

Almost Strongly b - δ -Continuous Function

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Abstract:

In this paper, we introduce and study two new of functions called almost strongly b - δ - continuous function by using the notions of b - δ -open sets and b - δ -closed sets.

Keywords: δ - open sets, b -open set, b - δ -open sets, strongly b - δ -open function, almost strongly b - δ -open function.

Introduction

Generalized open sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in general topology and real analysis concerns the various modified forms of continuity, separation axioms etc., by utilizing generalized open sets. In 1961, Levine^[12] introduced the concept of weak continuity as a generalization of continuity. Later in 1963, Levine^[13] also introduced the concept of semi open sets in topological space. Since then numerous applications have been found in studying different types of continuous like maps and separation of axioms.. Hussain^[10] introduced almost continuity as another generalization of continuity and Andrew and Whittlesy^[1] introduced the concept of closure continuity which is stronger than weak continuity. The concept of δ - interior, δ -closure, θ -interior and θ -closure operators were first introduced by Velico^[25] in 1968, for the purpose of studying the important class of H -closed spaces. These operators have since been studied intensively by many authors. The collection of all δ -open sets in a topological space (X, τ) forms a topology.

In 1970, Levine^[14] initiated the study of generalized closed sets, i.e., the sets whose closure belongs to every open superset and defined the notation of $T_{1/2}$ space to be one in which the closed sets and generalized closed sets coincide. In 1980, the notion of δ -continuous function was introduced and studied by Noiri^[19]. Later in 1982, Mashhour.et.al.,^[15] introduced the concept of pre open sets. In 1986, the notion of semi-pre open set was introduced by Andrijevic^[2]. Later in 1996, Andrijevic^[3] introduced a class of generalized open sets in a topological space, so called b -open sets. The class of b -open sets is contained in the class of semi preopen sets and contains all semi-open sets and pre-open sets. In 2003, Ganguly.et al.^[9] introduced the notion of strongly δ -continuous function in topological spaces. Later, El.Atik [8] introduced and studied the notion of b -continuous function. He also introduced and studied a new class of functions called b -irresolute function. This notion has been studied extensively in recent years by many topologists. In 2013 the notions of b - δ -closed Sets was introduced and studied by Padmanaban^[22]. Later in 2015, the concept of b - δ -continuous function^[5] and Strongly b - δ -continuous function^[4] was introduced by S.Anuradha and S.Padmanaban. The purpose of this paper is to introduce and investigate the function of almost strongly b - δ - continuous function. We investigate some of the fundamental properties of this class of functions. We recall some basic definitions and known results.

PRELIMINARY

Throughout this paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space (X, τ) . We denote closure and interior of A by $\text{cl}(A)$ and $\text{int}(A)$, respectively. The set A is said to be regular open (resp. regular closed) [24] if $A = \text{int}(\text{cl}(A))$ (resp. $A = \text{cl}(\text{int}(A))$). The family of all regular open (resp. Regular closed) sets of (X, τ) is written by $\text{RO}(X, \tau)$ (resp. $\text{RC}(X, \tau)$). This family is closed under the finite intersections (resp. Finite unions). The δ -closure of A [25] is the set of all x in X such that the interior of every closed neighbourhood of x intersects A non trivially. The δ -closure of A is denoted by $\text{cl}_\delta(A)$ or $\delta\text{-cl}(A)$. The δ -interior of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\delta\text{-int}(A)$. The subset A is called δ -open if $A = \delta\text{-int}(A)$. i.e, a set is δ -open if it is the union of regular open sets. The complement of δ -open set is δ -closed. Alternatively, a set $A \subset (X, \tau)$ is called δ -closed if $A = \delta\text{-cl}(A)$, where $\delta\text{-cl}(A) = \{x \in X : \text{int}(\text{cl}(U)) \cap A \neq \emptyset, U \in \tau \text{ and } x \in U\}$.

A subset A is said to be b -open [3] if $A \subset \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$. The complement of b -open is said to be b -closed. The intersection of b -closed sets of X containing is called b -closure of A and denoted by $b\text{-cl}(A)$. The union of all b -open sets of X contained in A is called b -interior and is denoted by $b\text{-int}(A)$. The subset A is b -regular if it is b -open and b -closed. The family of b -open (b -closed, b -regular) sets of X is denoted by $\text{BO}(X)$ (resp. $\text{BC}(X), \text{BR}(X)$) and family of all b -open (b -regular) sets of X containing a point $x \in X$ is denoted by $\text{BO}(X, x)$ (resp. $\text{BR}(X, x)$).

Let A be a subset of a topological space (X, τ) . A point x of X is called a b - δ -cluster point [22] of A if $\text{int}(b\text{-cl}(U)) \cap A \neq \emptyset$ for every b -open set U of X containing x . The set of all b - δ -cluster point of A is called b - δ -closure of A and is denoted by $b\text{-}\delta\text{-cl}(A)$. A subset A of a topological space (X, τ) is said to be b - δ -closed, if $A = b\text{-}\delta\text{-cl}(A)$. The complement of a b - δ -closed set is said to be b - δ -open set. The b - δ -interior of a subset A of X is defined as the union of all b - δ -open sets contained in A and is denoted by $b\text{-}\delta\text{-int}(A)$. Alternatively, a point x in X is called b - δ -interior point of A , if there exists a b -open sets containing x such that $\text{int}(b\text{-cl}(U)) \subseteq A$. The set of all b - δ -interior points of A is called b - δ -interior of A . The family of all b - δ -open sets of the space (X, τ) is denoted by $B\delta\text{O}(X)$ and the family of all b - δ -closed sets of the space (X, τ) is denoted by $B\delta\text{C}(X)$. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be b - δ -continuous (briefly b - δ -c) if for each $x \in X$ and each open set V of (Y, σ) containing $f(x)$, there exists a b -open set U in (X, τ) containing x such that $f(\text{int}(b\text{-cl}(U))) \subseteq V$.

A topological space (X, τ) is said to be

Almost regular [23] if for any regular open set $U \subseteq X$ and each point $x \in U$, there is a regular open set V of X such that $x \in V \subseteq \text{cl}(V) \subseteq U$,

Almost b -regular [21] if for any regular closed set $F \subseteq X$ and any point $x \in X - F$, there exist disjoint b -open sets U and V such that with $x \in U$ and $F \subseteq V$,

Semi-regular [16] if for each semi-closed $A \subseteq X$ and each point $x \notin A$ there exist disjoint semi-open sets $U, V \subseteq X$ such that $x \in U$ and $A \subseteq V$,

Hausdorff [21] if and only if $\{x\} = \bigcap \{\text{cl}(V) : x \in V \in \tau\}$ for each $x \in X$.

Frontier [7] of a non empty subset A of the space (X, τ) is the set of all elements x of X , such that each neighborhood of x contains elements of both A and its complement $X - A$.

rT_0 [2] if for any pair of distinct points of X , there exists a regular open set containing one of the points but not other,

b - T_2 [21], if for each pair of distinct points x and y in (X, τ) , there exists $U \in \text{BO}(X, x)$ and $V \in \text{BO}(X, y)$ such that $U \cap V = \emptyset$. i.e., If every two distinct points of (X, τ) can be separated by disjoint b -open sets.

A nonempty subset A of a topological space (X, τ) is said to be

b-closed relative to X [21] if for every cover $\{V_\alpha : \alpha \in I\}$ of A by b-open sets of (X, τ) , there exists a finite subset I_0 of I such that $A \subseteq \bigcup \{b\text{-cl}(V_\alpha) : \alpha \in I_0\}$.

N-closed relative to X [21] if for every cover $\{V_\alpha : \alpha \in I\}$ of A by regular open sets of (X, τ) , there exists a finite subset I_0 of I such that $A \subseteq \bigcup \{V_\alpha : \alpha \in I_0\}$.

Filter-base [17] on a set X if it satisfies the following conditions.

for any $x, y \in A$, there exists $B \in \mathcal{A}$, such that $B \subseteq x \cap y \in A$,

$\emptyset \notin \mathcal{A}$.

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be

Open [11], if $f(U)$ is open in (Y, σ) for every open set U of (X, τ) ,

b-open [8], if $f(U)$ is b-open in (Y, σ) for every open set U of (X, τ) ,

b-continuous [8], if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \text{BO}(X, \tau)$ such that $f(U) \subseteq V$,

δ -continuous function [19], if for each $x \in X$ and each open set V containing $f(x)$, there is an open set U containing x in (X, τ) such that $f(\text{int}(\text{cl}(U))) \subseteq \text{int}(\text{cl}(V))$,

be b- δ -continuous [5] (briefly b- δ -c), if for each $x \in X$ and each open set V of (Y, σ) containing $f(x)$, there exists a b-open set U in (X, τ) containing x such that $f(\text{int}(b\text{-cl}(U))) \subseteq \text{cl}(V)$.

Strongly continuous [11], if for every subset A of topological space (X, τ) , $f(\text{cl}(A)) \subseteq f(A)$,

Strongly δ -continuous [9], at a point $x \in X$ if and only if for any open neighbourhood V of $f(x)$ in (Y, σ) , there exists a δ -open neighbourhood U of x in (X, τ) such that $f(U) \subseteq V$,

Strongly b- δ -continuous [4] if for each $x \in X$ and each open set V of (Y, σ) containing $f(x)$, there exists a b-open set U in (X, τ) containing x such that $f(\text{int}(b\text{-cl}(U))) \subseteq V$,

R-map [6] if the preimage of every regular open subset of (Y, σ) is a regular open subset of (X, τ) .

A filter base F is said to be δ -convergent [19] to a point $x \in X$, if for any open set U containing x , there exists $B \in F$ such that $B \subseteq \text{int}(\text{cl}(U))$.

Lemma 2.1. [18] In a topological space (X, τ) ,

The intersection of an open set and a b-open set is b-open set,

The intersection of an α -open set and a b-open set is b-open set.

Lemma 2.2. [21] For a topological space (X, τ) , then the following are equivalent:

X is b-regular,

For each point $x \in X$ and for each open set U of (X, τ) containing x , there exists $V \in \text{BO}(X)$ such that $x \in V \subseteq b\text{-cl}(V) \subseteq U$,

For each subset A of X and each closed set F such that $A \cap F = \emptyset$, there exist disjoint $U, V \in \text{BO}(X)$ such that $A \cap U \neq \emptyset$ and $F \subseteq V$,

For each closed set F of X , $F = \bigcap \{b\text{-cl}(V) : F \subseteq V \text{ and } V \in \text{BO}(X)\}$.

Almost strongly b- δ -continuous function

The **Definition 3.1.** A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost strongly b- δ -continuous (briefly a.st.b- δ -c) if for each $x \in X$ and each open set V of (Y, σ) containing $f(x)$, there exists a b-open set U in (X, τ) containing x such that $f(\text{int}(b\text{-cl}(U))) \subseteq \text{int}(\text{cl}(V))$.

Theorem 3.2. 1. Every strongly b- δ -continuous function is almost strongly b- δ -continuous function,
2. Every almost strongly b- δ -continuous function is b- δ -continuous function.

Proof. Follows from definitions of strongly b - δ -continuous function and almost strongly b - δ -continuous function

The converse of above theorem need not be true as shown in following example.

Example 3.3. Let $x = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$. Define a function $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then we have $BO(X) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$.

Then f is almost strongly b - δ -continuous function but not strongly b - δ -continuous function, since for $V = \{a, c\}$ there exists no $U \in BO(X, x)$ for $x = c$ such that $f(\text{int}(b\text{-cl}(U))) \subseteq V$.

Example 3.4. Let $x = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Define a function $f: (X, \tau) \rightarrow (X, \sigma)$ by $f(a) = b$, $f(b) = c$ and $f(c) = c$. Then we have $BO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Then f is b - δ -continuous function but not almost strongly b - δ -continuous function, since for $V = \{b\}$ there exists no $U \in BO(X, x)$ for $x = a$ such that $f(\text{int}(b\text{-cl}(U))) \subseteq \text{int}(cl(V))$.

Theorem 3.5. Every strongly b - δ -continuous function is b - δ -continuous function.

Proof: Follows from Theorem 3.2.

Remark. From the above discussions we have the following diagram. None of the below implications are reversible.

Strongly b -continuous function \rightarrow Almost strongly b - δ -continuous function \rightarrow b - δ -continuous function

Definition 3.6. A filter base F is said to be b - δ -convergent to a point x in X , if for any b - δ -open set U containing x , there exists $B \in F$ such that $B \subseteq U$.

Theorem 3.7. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost strongly b - δ -continuous if and only if for each point $x \in X$ and each filter base F in X b - δ -converging to x , the filter base $f(F)$ is δ -convergent to x .

Proof. Suppose that $x \in X$ and F is any filter base in (X, τ) , b - δ -converging to x . By hypothesis, for any regular open set V in (Y, σ) containing $f(x)$, there exists $U \in B\delta O(X)$ containing x such that $f(U) \subseteq V$. Since F is b - δ -convergent to x in X , there exists $B \in F$ such that $B \subseteq U$. It follows that $f(B) \subseteq V$. Thus $f(F)$ is δ -convergent to $f(x)$.

Conversely, let x be a point in X and V be any regular open set containing $f(x)$. If we set $F = \{U \in B\delta O(X) : x \in U\}$, then F will be a filter base which b - δ -converges to x . So there exists U in F such that $f(U) \subseteq V$. This completes the proof.

Definition 3.8. Let (X, τ) be a topological space and A be a subset of (X, τ) . The b - δ -frontier of A is denoted by b - δ -Fr(A) and defined as b - δ -Fr(A) = b - δ -cl(A) \cap b - δ -cl($X - A$) = $(b$ - δ -cl(A)) - $(b$ - δ -int(A)).

Theorem 3.9. The set of all points $x \in X$ at which a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is not almost strongly b - δ -continuous is identical with the union of the b - δ -frontier of the inverse images of regular open sets of (Y, σ) containing $f(x)$.

Proof. Suppose that f is not almost strongly b - δ -continuous at $x \in X$. Then there exist no $U \in BO(X, x)$ such that $f(\text{int}(b\text{-cl}(U))) \subseteq V$ for each open set V of (Y, σ) containing $f(x)$. Then $\text{int}(b\text{-cl}(U)) \not\subseteq f^{-1}(V) \Rightarrow \text{int}(b\text{-cl}(U)) \subseteq X - f^{-1}(V) \Rightarrow \text{int}(b\text{-cl}(U)) \cap (X - f^{-1}(V)) \neq \emptyset$.

$x \in b$ - δ -cl($X - f^{-1}(V)$) = $X - (b$ - δ -int($f^{-1}(V)$)).

ie, $x \notin b$ - δ -int($f^{-1}(V)$).

On the otherhand, $x \in f^{-1}(V) \subseteq b$ - δ -cl($f^{-1}(V)$). Hence $x \in b$ - δ -cl($f^{-1}(V)$) - $(b$ - δ -int($f^{-1}(V)$)) and so $x \in b$ - δ -Fr[V]. Conversely, suppose that f is almost strongly b - δ -continuous at $x \in X$ and let V be any open set in (Y, σ) containing $f(x)$. Then, we have $f^{-1}(V)$ is b - δ -open and $x \in f^{-1}(V)$. Therefore, $x \notin b$ - δ -Fr[$f^{-1}(V)$] for each regular open sets V containing $f(x)$. This completes the proof.

Theorem 3.10. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function. Then the following hold:

1. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is b -continuous and (Y, σ) is almost regular, then f is almost strongly b - δ -continuous,
2. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost strongly b - δ -continuous and (Y, σ) is semi regular, then f is strongly b - δ -continuous.

Proof. 1. Let $x \in X$ and V be any open set in (Y, σ) containing $f(x)$. Since (Y, σ) is almost regular, there exists a regular open set W such that $f(x) \in W \subseteq \text{cl}(W) \subseteq V$. Since f is b -continuous, there exists $U \in \text{BO}(X)$ containing x such that $f(U) \subseteq W$. Suppose $y \notin \text{cl}(W)$. Then there exists an open neighbourhood G of y such that $G \cap W = \emptyset$. Since f is b -continuous, $f^{-1}(G) \in \text{BO}(X)$ and $f^{-1}(G) \cap U = \emptyset$. Hence $f^{-1}(G) \cap \text{int}(b\text{-cl}(U)) = \emptyset$. Therefore, we obtain $G \cap f(\text{int}(b\text{-cl}(U))) = \emptyset$ and $y \notin f(\text{int}(b\text{-cl}(U)))$. This shows that $f(\text{int}(b\text{-cl}(U))) \subseteq \text{cl}(W) \subseteq V$. Hence f is strongly b - δ -continuous function. By Theorem 3.2., f is almost strongly b - δ -continuous function.

2. Let $x \in X$ and V be any open set in (Y, σ) containing $f(x)$. Since (Y, σ) is semi regular, there exists a regular open set W such that $f(x) \in W \subseteq V$. Since f is almost strongly b - δ -continuous there exists $U \in \text{BO}(X)$ containing x such that $f(\text{int}(b\text{-cl}(U))) \subseteq W$. Therefore, we obtain $f(\text{int}(b\text{-cl}(U))) \subseteq V$. Hence f is strongly b - δ -continuous.

Theorem 3.11. Let (Y, σ) be a semi-regular space. Then $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost strongly b - δ -continuous if and only if f is strongly b - δ -continuous.

Proof. The proof follows from Theorem 3.10. and Theorem 3.2.

Theorem 3.12. If (X, τ) is almost b -regular then an R -map $f: (X, \tau) \rightarrow (X, \tau)$ is almost strongly b - δ -continuous.

Proof. Suppose that f is an R map and X is almost b -regular. Since f is an R -map, for any $x \in X$ and regular open set V containing $f(x)$, $f^{-1}(V)$ is regular open in x . Since X is almost b -regular, there exists $U \in \text{BO}(X)$ containing x such that $b\text{-cl}(U) \subseteq f^{-1}(V)$. We have $f(b\text{-cl}(U)) \subseteq V$. Hence $f(\text{int}(b\text{-cl}(U))) \subseteq V$. This shows that f is strongly b - δ -continuous. By Theorem 3.2., f is almost strongly b - δ -continuous.

Theorem 3.13. If $f: (X, \tau) \rightarrow (X, \tau)$ is almost continuous and X is b -regular, then f is almost strongly b - δ -continuous.

Proof. Suppose that f is almost continuous and (X, τ) is b -regular. Then since f is almost continuous, for each $x \in X$ and any regular open set V of y containing $f(x)$, $f^{-1}(V)$ is open in (X, τ) and $x \in f^{-1}(V)$. Since X is b -regular, by Lemma 2.2., there exists $U \in \text{BO}(X)$ such that $x \in U \subseteq b\text{-cl}(U) \subseteq f^{-1}(V)$. Therefore, we have $f(\text{int}(b\text{-cl}(U))) \subseteq V$. By Theorem 3.2, f is almost strongly b - δ -continuous.

Theorem 3.14. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an almost strongly b - δ -continuous injection and (Y, σ) is rT_0 , then X is $b\text{-}T_2$.

Proof. Let $x_1, x_2 \in X$ and $x_1 \neq x_2$. Since f is injective and since (Y, σ) is rT_0 , $f(x_1) \neq f(x_2)$ and there exists a regular open set V containing $f(x_1)$ but not containing $f(x_2)$ or a regular open set W containing $f(x_2)$ but not containing $f(x_1)$. Consider the first case holds. Then since f is almost strongly b - δ -continuous, there exists $U \in \text{BO}(X)$ containing x_1 , such that $f(\text{int}(b\text{-cl}(U))) \subseteq V$. Therefore, we obtain $f(x_2) \notin f(\text{int}(b\text{-cl}(U)))$ and $x_2 \notin X - \text{int}(b\text{-cl}(U))$. Hence (X, τ) is $b\text{-}T_2$.

Theorem 3.15. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an almost strongly b - δ -continuous injection and (Y, σ) is Hausdorff, then X is $b\text{-}T_2$.

Proof. Every Hausdorff space is rT_0 . The proof follows from Theorem 3.14.

Theorem 3.16. Let $f, g: (X, \tau) \rightarrow (Y, \sigma)$ be functions and (Y, σ) be a Hausdorff space. If f is almost strongly b - δ -continuous and g is an R -map, then the set $A = \{x \in X: f(x) = g(x)\}$ is b -closed in (X, τ) .

Proof. Let $x \notin A$. Then $f(x) \neq g(x)$. Since (Y, σ) is Hausdorff, there exist open sets V_1 and V_2 in (Y, σ) such that $f(x) \in V_1$ and $g(x) \in V_2$ and $V_1 \cap V_2 = \emptyset$; hence $\text{int}(\text{cl}(V_1)) \cap \text{int}(\text{cl}(V_2)) = \emptyset$. Since f is almost strongly b - δ -continuous, there exists $G \in \text{BO}(X)$ containing x such that $f(\text{int}(b\text{-cl}(G))) \subseteq \text{int}(\text{cl}(V_1))$. Since g is an R -map, $g^{-1}(\text{int}(\text{cl}(V_2)))$ is regular open in (X, τ) and $x \in g^{-1}(\text{int}(\text{cl}(V_2)))$.

Put $U = G \cap g^{-1}(\text{int}(\text{cl}(V_2)))$. Since $G \in \text{B}\delta\text{O}(X) \Rightarrow G \in \text{BO}(X)$, by Lemma 2.1., $x \in U \in \text{BO}(X)$ and $U \cap A = \emptyset$. Hence we have $x \notin b\text{-cl}(A)$. Thus $b\text{-cl}(A) \subseteq A$. But $A \subseteq b\text{-cl}(A)$. Hence $A = b\text{-cl}(A)$ which implies A is b -closed.

Theorem 3.17. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost strongly b - δ -continuous and (Y, σ) is Hausdorff, then $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$ is b - δ -closed on the product space $X \times X$.

Proof: Let $(x_1, x_2) \notin A$. Then $f(x_1) \neq f(x_2)$. Since (Y, σ) is Hausdorff, there exist open sets V_1 , and V_2 such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$, hence $\text{int}(\text{cl}(V_1)) \cap \text{int}(\text{cl}(V_2)) = \emptyset$. Since f is almost strongly b - δ -continuous, there exists $U_1 \in \text{BO}(X)$ containing x_1 such that $f(\text{int}(b\text{-cl}(U_1))) \subseteq \text{int}(\text{cl}(V_1))$ and there exists $U_2 \in \text{BO}(X)$ containing x_2 such that $f(\text{int}(b\text{-cl}(U_2))) \subseteq \text{int}(\text{cl}(V_2))$. Set $D = \text{int}(b\text{-cl}(U_1)) \times \text{int}(b\text{-cl}(U_2))$. It follows that $(x_1, x_2) \in D \in \text{BR}(X \times X)$ and $D \cap A = \emptyset$. Hence we have $x \notin b\text{-}\delta\text{-cl}(A)$.

Theorem 3.18. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an almost strongly b - δ -continuous function and A is b -closed relative to X , then $f(A)$ is N -closed relative to (Y, σ) .

Obvious.

Corollary 3.19. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an almost strongly b - δ -continuous surjection. Then the following are hold:

1. If (X, τ) is b -closed, then (Y, σ) is nearly compact,
2. If (X, τ) is countable b -closed, then (Y, σ) is nearly countable compact.

Proof. Follows from the definitions.

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