[Volume 3 issue 8 August 2015]
PageNo.1106-1109 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Convergence Classes

***Basma Al-Shutnawi* [1], *Mohammad Zannon* [2]**

[1,2]Department of Mathematics, Tafila technical university, P.O. Box 179 , Tafila 66110, Jordan.

E-mail -[1] basma@ttu.edu.jo

E-mail-[2]zanno1ms@gmail.com

Abstract

In this paper we present the class of convergent set for the formal power series $f(z,t)=\sum_{j=0}^{\infty} P_j(z)t^j$, where every $P_j$is a polynomial whose degree is bounded by a linear function $\deg P_j \leq Aj+B$, for some $A>0, and\ B \geq 0$. The class $C(\delta,A,B)$, and the$C(\delta,A,B)$convergent set are considered, we find equalconvergent classes.Where the equal convergent classes contains the same elements.

Introduction

The purpose of this paper is to introduce a set in$\mathbb{C}$, describe the convergence of the formal series .We study convergence sets of formal power series of the type$f(z,t)=\sum_{j=0}^{\infty} P_j(z)t^j$, where $P_j(z)$are polynomials with $\deg P_j \leq j$ as in [1],[2] and [3],[4]. We say that $f(z,t)$ is convergent if there exist a constant C such that

$$|P_j(z)| \leq C.$$

The classical theorem of hartog say that the formal power series is convergent if the series convergent when restricted to every line through the origin as in [9],[7],[6],[8],[5]. An interpretation of this theorem is that $f$is holomorphic in $\mathbb{C}^n$if for each axis f is holomorphic on every complex line parallel to this axis. This interpretation leads to a number of question described in the article by K. Spalk, P. Tworzewki , T. Winiarski as in [9] in the following way: Osgood-Hartogs-Type problems ask for properties of object whose

restrictions to certain test-sets are well known. A revision of the Hartogs's theorem states that a series f converges if and only if it converges along alldirections $\zeta \epsilon \square^{n-1}$. On the contrary for a divergent series it is still possible converge in some directions, so it is natural to consider what the set of all such directions is.Lelong in [5], showed that a formal power series $g(x,y)$ converges in some neighborhood of the origin if there exists a set $E \subset \mathbb{C}$of positive capacity such that, for each $s \in E$the formal power series

$g(x, sx)$converges in some neighborhood of the origin (of a size possibly depending on s).[Bochnak 1970; Siciak 1970] in [10], Proved the following theorem. Let $f \in C^{\infty}(D)$, where $D$is a domain in $\mathbb{R}^n$ containing 0. Suppose $f$is analytic on every segment through 0. Then $f$is analytic in a neighborhood of 0 (as a function of $n$variables). Abyankar and Moh (see [11]), showed that the test sets in many cases form a family of linear subspaces of lower dimension. For example, articles by S.S. Abhyankar, T. T. Moh[11], N. Levenberg and R. E. Molzon [6] , A. Sathaye [8], M. A. and others consider the convergence of formal power series of several variables provided the restriction of such a series on each element of a sufficiently large family of linear subspaces is convergence. T. S. Neelon [12] proved that a formal power series is convergence if its restriction to certain families of curves or surfaces parametrized by polynomial maps are convergence.Fridman and Ma in [13] considered two families of test sets separately.Fridman, Ma and T. S. Neelon[4] generalized the result of P. Lelong and A. Sathaye for the linear case by introducing the family ofanalytic curves as the subspace substitute.We say that that a series $f(z,t) = \sum P_n(z)t^n$ is of class $C(\delta, A, B)$ if deg$P_n \leq An^{\delta-1} + B$ for n sufficiently large, and for$\delta \geq 1, A > 0, B \geq 0$. We definea$C(\delta, A, B)$ convergence set $E \subset \mathbb{C}$,if there exists an $f \in C(\delta, A, B)$ such that $E = Conv(f)$, where $Conv(f)$is the convergent set *i.e* the set of complex number z in $\mathbb{C}$ such that $f(z,t)$ converges in t see [3]. We finding equal convergence sets in the following theorem.

Let $[X]$ denote the greatest integer that is greater than or equal to X.

Theorem: For any fixed $\delta \geq 0$, every $C(\delta, A, B)$ convergence set is a $C(\delta, 1, 0.5)$ convergence set.

Proof. Let E be a $C(\delta, A, B)$ convergence set. Then there exist an $f \in C(\delta, A, B)$,

$$f(z,t) = P_0(z) + P_1(z)t + \cdots + P_n(z)t^n + \cdots,$$

With $E = Conv(f)$ and deg $P_n \leq An^{\delta-1} + B$.

Let

$$g(z,t) = \sum_{j=0}^{\infty} P_j(z)t^{N_j},$$

Where $N_j = \left[(A+B)^{\frac{1}{\delta-1}}\right] j$. Then, for $j \geq 1$,

$$\deg P_j \le Aj^{\delta-1} + B$$

$$\le (A+B)j^{\delta-1}$$

$$= \left((A+B)^{\frac{1}{\delta-1}} .j\right)^{\delta-1}$$

$$\le \left(\left\lceil (A+B)^{\frac{1}{\delta-1}} \right\rceil j\right)^{\delta-1} + 0.5$$

$$= N_j^{\delta-1} + 0.5 .$$

Which implies that $g(z,t) \in C(\delta,1,0.5)$. Now we need to prove that $E = Conv(g)$. For $z \in E$ there exists a positive number c such that $|P_n(z)| < c^n$. Let $c_g = c^{1/\left\lceil (A+B)^{\frac{1}{\delta-1}} \right\rceil}$.

Then

$$|P_n(z)| < c_g^{\left\lceil (A+B)^{\frac{1}{\delta-1}} \right\rceil n} = c_g^{N_n},$$

Which shows that $z \in Conv(g)$, and $Conv(f) \subset Conv(g)$. Reverse the above steps we conclude that $Conv(g) \subset Conv(f)$. It's easy to see that the generalization for the previous theorem still true, $i.e$ For any fixed $\delta \ge 0$, every $C(\delta,A,B)$ convergence set is a $C(\delta,1,k)$ convergence set, where $k \ge 0$.

Conclusion:

in this paper we show that every $C(\delta,A,B)$ convergence set is a $C(\delta,A,k)$ convergence set. In the next paper we will look for new classes of convergence sets.

References:

1. J. Ribon, Holomorphic extensions of formal objects, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5), 3 (2004), 657-680
2. R. P´erez-Marco, A note on holomorphic extensions, preprint, 2000
3. B. Al-Shutnawi, "On convergence sets of formal power series", PhD dissertation, USA, 2013
4. B.L. Fridman, D. Ma, T.S. Neelon, "Nonlinear convergence sets of divergent power series", preprint.

DOI:10.18535/ijmcr/v3i8.01

5. P. Lelong, "On a problem of M.A. Zorn", Proc. Amer. Math. Soc., 2(1951), pp 11-19.
6. N. Levenberg, R.E. Molzon," Convergence sets of a formal power series", Math. Z., 197(1988), pp 411-420.
7. D. Ma, T.S. Neelon, "On convergence sets of formal power series", preprint.
8. Sathaye, "Convergence sets of divergent power series", J. ReineAngew. Math., 283(1976), 86-98.Kokyoroku in Math. 14, Tokyo, 1982.
9. K. Spallek, P. Tworzewski, T. Winiarski," Osgood-Hartogs-theorem of mixed type", Math. Ann,288(1990), pp 75-88.
10. J. Bochnak, "Analytic function in Banach spaces," Studia Math., 35(1970), pp.273--292.
11. S. S. Abhyankar and T. T. Moh, "A reduction theorem for divergent power series," J. ReineAngew. Math., 241(1970), pp.27-33.
12. T. S. Neelon, "Restrictions of power series and functions to algebraic surfaces,"Analysis(Munich), 29(2009), pp.1-15.
13. B. L. Fridman and D. Ma, "Osgood {Hartogs-type properties ofpower series and smooth functions}," Math.cv 251(2011), pp. 67-79.