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Solution of Boundary Layer Equations for Non-Newtonian Power Law Fluid past Flat Porous Surface.

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Abstract
Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid past flat surface has been considered. Theoretical analysis for the boundary layer flow of power law fluid past flat surface is considered. The governing equations of continuity and momentum are transformed into ordinary differential equations using similarity transformations. The equations are solved by using method of successive approximations starting with zeroth approximation. Local Skin Friction Coefficient
$c_f{}^* = [f_1{}''(0)]^{\ n}$ has been calculated. Velocity Profiles have been drawn for various values of n and suction /injection parameter f_w.

Keywords
boundary layer, power law fluids, successive approximations, flat surface, Skin friction, suction /injection.

Introduction
The theoretical analysis of an external boundary layer flow of non-Newtonian fluid was first performed by Schowalter [1] and Acrivos and Shah [2].
A similarity solution to the boundary layer equations for a power fluid flowing along a flat plate were obtained by [2].The solution to boundary layer flow of non –Newtonian power law fluid past flat plate were obtained by Jadhav and Waghmode[3].The problems related to non-Newtonian fluids were considered by Zang et al[4]. Similarity Solutions to Non- Newtonian Power law Fluids were Obtained by Mohamed Guedda, Zakia Hammouch[6].Jadhav and Waghmode [5] has studied the problem of Heat transfer to non-Newtonian power law fluid past a continuously moving porous flat plate. In this paper we want study the effects of suction/injection parameter f_w.Flows of this type are encounted in glacial advance, in transport of coal slurries down conveyor belts and in several other geophysical and industrial contexts.

Mathematical Analysis

The problem considered here is the steady boundary layer flow of non-Newtonian power law fluid past flat surface. In the absence of body force, external pres-sure gradients and viscous dissipation, the laminar boundary layer equations
expressing conservation of mass and momentum are governed by the equations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \qquad \text{----- (1)}$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad \text{------ (2)}$$

With boundary conditions

$u = U$, v=$v_{0(x)}$ at y=0 and u$\rightarrow 0, v = 0 \; as \; y \rightarrow \infty$ ---- (3)

Two-Point Boundary Value Problems:-

We introduce dimensionless variables as follows,

$$x = X/L, \quad y = (NR_e)^{\frac{1}{n+1}} \; (Y/L),$$

$$u = U/U_w, \quad v = (NR_e)^{\frac{1}{n+1}} \; (V/U_w),$$

$$NR_e = \gamma {U_w}^{2-n} L^n \, , \qquad \text{-------------- (4)}$$

Where L is characteristic length and $U_w \geq 0$

Substituting (4) into equations (1)-(3) and introducing the stream function ψ(x, y) and similarity variable η as

$$\psi = Ax^{\propto} f(\eta), \qquad \eta = Bx^{\beta} \, y$$

Where A, B, ∝, β are constants to be determined, and $f(\eta)$denotes the dimensionless stream function. For existence of similarity solutions, we choose $u = \frac{\partial \psi}{\partial y} \;\; and \;\; v = -\frac{\partial \psi}{\partial x}$.

This leads to

$$\propto + \beta = 0, \qquad AB = 1$$

$$B = [(n+1)\gamma]^{\frac{-1}{n+1}}, \qquad \propto = \frac{1}{n+1}$$

-------- (5)

With this the equations (1)-(3) reduces to the form

$$n(n+1)(f'')^{n-1} f''' + f\,f'' = 0 \quad \text{------- (6)}$$

$$f(0) = f_w, \quad f'(0) = 1, \qquad f'(\infty) = 0 \quad \text{---- (7)}$$

Method of Solution

To solve the non-linear differential equation (8) under the boundary conditions (9), we use method of successive approximations starting with zeroth approximation.
For zeroth approximation, we assume

$$f(\eta) = f_w + \frac{1}{\beta} e^{-\beta\eta} - \frac{1}{\beta} \quad \text{------------ (8)}$$

Where β is arbitrary constant to be determined such that for the first approximation $f'(0) = 1$, i. e. β is real root of the equation

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$$\beta^{n+1} - \frac{(1+f_w\beta)}{n(n+1)(n-2)^2} + \frac{1}{n(n+1)(n-3)^2} = 0 \quad \text{--------------- (9)}$$

The different successive approximations can be obtained from

$$f'''_i = \frac{-1}{n(n+1)} [f_{i-1}(f_{i-1}'')^{2-n}] \quad \text{---- (10)}$$

For the first approximation, we have,

$$f_1''' = \frac{-1}{n(n+1)} [f_0(f_0'')^{2-n}] \quad \text{---------- (11)}$$

Integrating (13) with boundary conditions (9), we obtain

$$f_1''(\eta) = -A_1\ (n-2)\beta e^{(n-2)\beta\eta} + A_2(n-3)\beta e^{(n-3)\beta\eta}$$
$$f_1'(\eta) = -A_1\ e^{(n-2)\beta\eta} + A_2 e^{(n-3)\beta\eta} \quad \text{-------- (12)}$$

Where,

$$A_1 = \frac{(1+f_w\beta)}{n(n+1)(n-2)^2\beta^{n+1}}, \qquad A_2 = \frac{1}{n(n+1)(n-3)^2\beta^{n+1}} \quad \text{--------- (13)}$$

For different values of power law fluid index *n* skin friction coefficient $c_f^* = [f_1''(0)]^n = [-A_1\ (n-2)\beta + A_2(n-3)\beta]^n$ can be calculated.

Calculation of Skin friction c_f^*

n/f_w	0	0.2	0.5	0.8	1.0	-0.2	-.5	-.8	-1.0
0.4	.7845	.8600	.9817	1.1170	1.2164	.7117	.6565	.4911	.4060
1.0	.4082	.4763	.5833	.6961	.7743	.3431	.2500	.1656	.1228
1.4	.2568	.3154	.4094	.5083	.5767	.2009	.1248	.0505	.0259

Conclusions

For fixed value of suction /injection parameter f_w, as *n* increases skin friction and Velocity decreases.

For fixed value of n as suction increases the skin friction and velocity increases while as injection increases the skin friction and velocity decreases.

For n=1 the results tallies with the results for Newtonian fluid.

For $f_w = 0$ the results tallies with the results obtained by Jadhav[5]

Velocity Profiles drawn for n = 0.4, 1.0, 1.4 shows the behavior of power law fluids.

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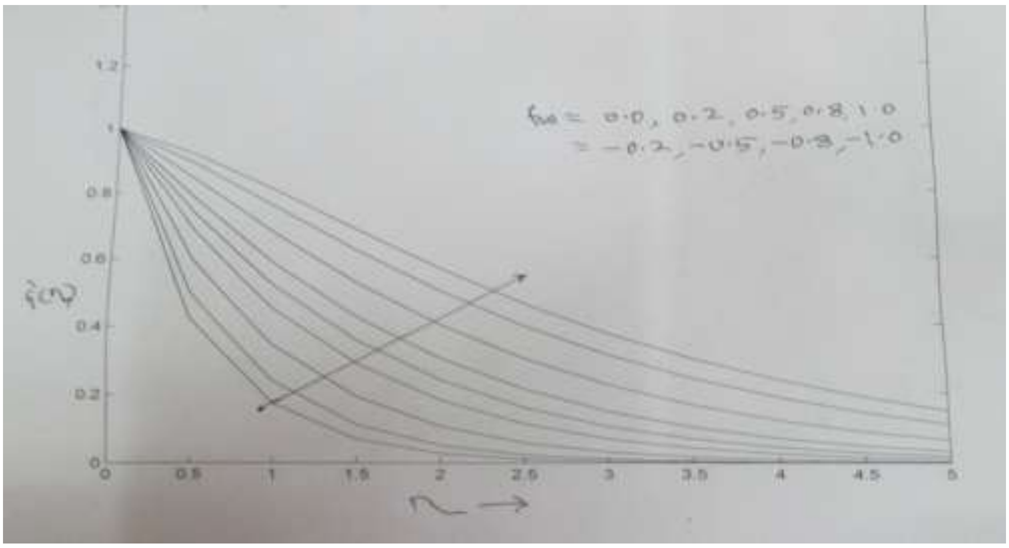
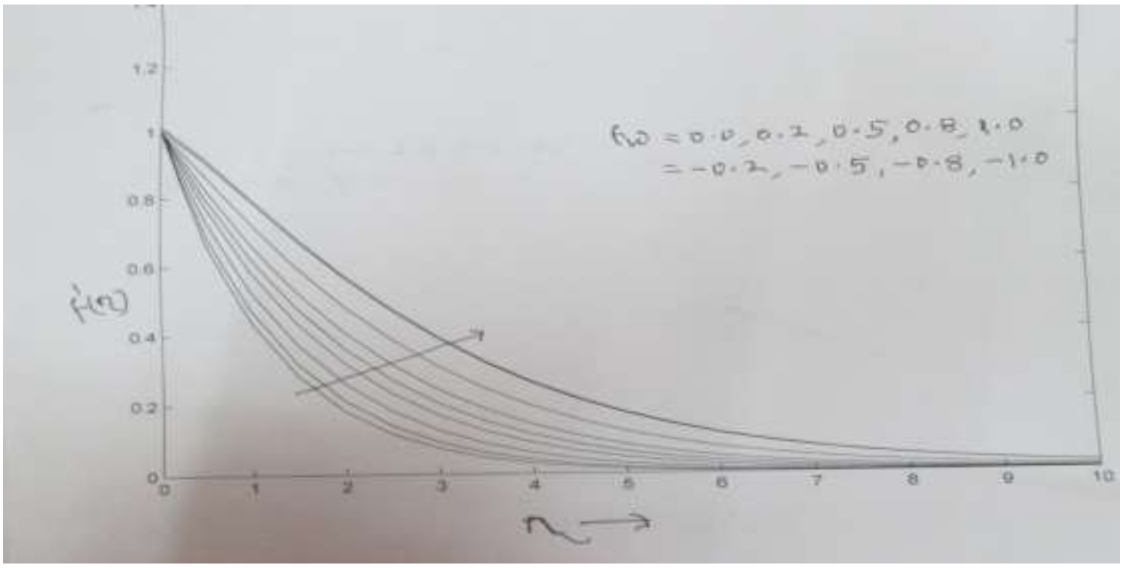
Velocity Profiles for n = 0.4 and various values of suction/injection f_w



Velocity Profiles for $n = 1.0$ and various values of suction/injection f_w



Velocity Profiles for $n = 1.4$ and various values of suction/injection f_w