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Uniqueness Of Meromorphic Functions Sharing One Value And Having A Same A-Point

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ABSTRACT : In this paper we prove a uniqueness theorem for a meromorp function which is sharing one value and a small meromorphic function with its derivative.

Definition: If two meromorphic functions f and g have the same a-point with the same multiplicities, we denote it by $E(a,f) = E(a,g)$

Our main result is the following.

Theorem: Let f be a non constant meromorphic function with $E(\infty, f) = E(\infty, f')$, $E(a, f') = E(b, f'')$ and satisfying the differential equation $kf'' - f' + (a - kb) = 0$,

(1)

where a, b and k are non zero constants.

If $N(r,f) + N\left(r, \frac{1}{f}\right) = S(r,f)$, then $bf' = af''$. Further, if there exist complex numbers c, d such that $\overline{N}\left(r, \frac{1}{f-c}\right) + \overline{N}\left(r, \frac{1}{f'-d}\right) + \overline{N}(r,f) = S(r,f)$,

then,
$$\frac{f-c}{a} = \frac{f'-d}{b}$$

To prove the above result, we require the following Lemmas.

Lemma 1 [2] Let f_1, f_2 be non constant meromorphic functions such that $af_1 + bf_2 \equiv 1$ where a, b are non zero constants.

Then, $T(r,f_1) < \overline{N}\left(r,\frac{1}{f_1}\right) + \overline{N}\left(r,\frac{1}{f_2}\right) + \overline{N}(r,f_1) + S(r,f_1).$

Lemma 2[1]: Let f be a non constant meromorphic function.

Then, for $n \geq 1$,

$$N\left(r,\frac{1}{f^{(n)}}\right) \leq 2^{n-1}\left[\overline{N}\left(r,\frac{1}{f}\right) + \overline{N}(r,f)\right] + N\left(r,\frac{1}{f}\right) + S(r,f).$$

Lemma 3: Let f be a non constant meromorphic function.

Then, $N\left(r,\frac{1}{f'}\right) + N\left(r,\frac{1}{f''}\right) \leq 3\overline{N}\left(r,\frac{1}{f}\right) + 3\overline{N}(r,f) + 2N\left(r,\frac{1}{f}\right)$

The proof is omitted as it directly follows from Lemma 2.

Proof of the Theorem

From (1), we have $kf'' - f' + (a - kb) = 0$

Therefore $\frac{f' - a}{f'' - b} = k$ (2)

where a, b and k are non zero constants .

If $k \neq \frac{a}{b}$, then $\frac{f'}{a - bk} - \frac{kf''}{a - bk} \equiv 1$

Then, by Lemma 1,

$$T(r,f') < \overline{N}\left(r,\frac{1}{f'}\right) + \overline{N}\left(r,\frac{1}{f''}\right) + \overline{N}(r,f') + S(r,f')$$

Using Lemma 3 and noting that $S(r,f') = o\{T(r,f')\}$

$$= o\{T(r,f)\}$$

$$= S(r,f),$$

we get, $T(r, f') \le 3\overline{N}\left(r,\frac{1}{f}\right) + 3\overline{N}(r,f) + 2N\left(r,\frac{1}{f}\right) + N(r,f) + \overline{N}(r,f) + S(r,f)$

Thus , $T(r,f') \le 5N\left(r,\frac{1}{f}\right) + 5N(r,f) + S(r,f)$

Therefore, $T(r,f') \le 5\left[N(r,f) + N\left(r,\frac{1}{f}\right)\right] + S(r,f)$

Or , $T(r,f') \le S(r,f)$, using hypothesis.

Hence, $1 \le \frac{S(r,f)}{T(r,f')} = \frac{S(r,f)}{O\{T(r,f)\}} \to 0$ as $r \to \infty$

which is a contradiction.

This contradiction proves that $k = \frac{a}{b}$

If $k = \frac{a}{b}$, (2) becomes $\frac{f'-a}{f''-b} = \frac{a}{b}$

Therefore, $bf' - ab = af'' - ab$

Or, $bf' = af''$

Further, integrating, we get

$$f = \frac{a}{b} f' + x \quad \text{where} \quad x \quad \text{is a constant.}$$

Therefore, $f - c = \frac{a}{b}(f'-d) + (x-c) + \frac{a}{b}d$

Therefore, $f - c = \frac{a}{b}(f'-d) + x_1$ where $x_1 = x - c + \frac{a}{b}d$

If $x_1 = x - c + \frac{a}{b}d \ne 0$, then by the Second Fundamental Theorem, we have

$$T(r,f) \le T(r,f-c) + O(1)$$

$$\le \overline{N}(r,f) + \overline{N}\left(r,\frac{1}{f-c}\right) + \overline{N}\left(r,\frac{1}{f-c-t_1}\right) + S(r,f).$$

Therefore, $T(r,f) \le \overline{N}(r,f) + \overline{N}\left(r,\frac{1}{f-c}\right) + \overline{N}\left(r,\frac{1}{f'-d}\right) + S(r,f)$

$< S(r, f)$, using hypothesis,

which is a contradiction.

This contradiction proves that $x_1 = 0$

Therefore, $f - c = \frac{a}{b}(f' - d)$

Therefore, $\frac{f-c}{a} = \frac{f'-d}{b}$,

Hence the result.

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